RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2017

FIRST YEAR (BATCH 2017-20)

PHYSICS (Honours)

Date : 12/12/2017

Time : 11.00 am – 3.00 pm

Paper : I

Full Marks : 100

[Use a separate Answer Book for <u>each group</u>]

Group – A

(***Answer any four questions***) [4×10]

1. a) Evaluate $\oint (x^2-2xy)dx+(x^2y+3)dy$ around the boundary of the region defined by $y^2=8x$ and $x = 2$ (i) directly and (ii) by using Green's theorem. [3+2]

   b) Suppose $\vec{F}=2y\hat{i}-z\hat{j}+x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2=8x$ in the first octant bounded by the planes y = 4 and z = 6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$. [5]

2. a) By drawing a figure, geometrically show that $\vec{r}.d\vec{r}=rdr$. [3]

   b) Show that the line integral of $\vec{F}=-\hat{i}y+\hat{j}x$ around a closed curve in XY plane is twice the area enclosed by curve. [3]

   c) Verify stokes' theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ where S is the upper half surace of the sphere $x^2+y^2+z^2=1$ and C is its boundary. [4]

3. a) Express $\vec{\nabla}\cdot\vec{A}$ in orthogonal co-ordinates. [3]

   b) Find the volume element dv in (i) spherical and (ii) cylindrical co-ordinates. [2+2]

   c) Given that $\vec{B}=(x^2+2xz)\hat{i}+2yz\hat{j}-(z^2+2zx)\hat{k}$ Find a vector $\vec{A}$ such that $\vec{B}=\vec{\nabla}\times\vec{A}$. Is the solution of $\vec{A}$ unique? [3]

4. a) Prove that $\vec{\nabla}\cdot(\vec{A}\times\vec{r})=\vec{r}\cdot(\vec{\nabla}\times\vec{A})$. [3]

   b) Define a function
   $$R_\sigma(x) = \frac{1}{2\sigma} \quad \text{for } (a-\sigma)<x<(a+\sigma)$$
   $$= 0 \quad \text{otherwise}$$
   Find $\int_{-\infty}^{+\infty} R_\sigma(x)dx$. Under what condition $R_\sigma(x)$ represents Dirac-delta function. [2+1]

   c) Show that $\delta(nx)=\frac{1}{|n|}\delta(x)$. Using this relation show that delta function is an even function. [3+1]

5. a) Solve the following equation by the method of separation of variables. $3\frac{\partial u(x,y)}{\partial x}+2\frac{\partial u(x,y)}{\partial y}=0$ given that $u(x,0)=4e^{-x}$. [4]

   b) Apply the method of separation of variables to obtain a formal solution $u(x,t)$ of the problem which consists of the heat equation $\alpha_1^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$,

   the boundary conditions : $u(0,t)=0 \quad t>0$

   $u(2,t)=0 \quad t>0$

   and the initial condition $u(x,0)=100\sin x,\ 0\le x\le 2$. [6]

6. a) Discuss about the regular/essential singular point of the following equation $(1-x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\lambda y=0$. [3]

b) Use the method of Frobenius to find solutions of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$. [7]

7. a) Expand $f(x)=x^2$ for $-\pi \le x \le \pi$ in a fourier series. [3]

b) Draw the graphical representation of $f(x)=x^2$ in $[-\pi,\pi]$ and based on Dirichlet's condition draw its periodic extension (i.e outside of $[-\pi,\pi]$) [2]

c) Show that $\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}$. (use the answer in (a)). [3]

d) Find the Fourier transform of the function $f(x)=\begin{cases}1 & |x|<a \\ 0 & |x|>a\end{cases}$. [2]

<u>Group – B</u>

(*Answer <u>any three</u> questions*) [3×10]

8. Consider a particle moving in the xy plane under the force $\vec{F}(r)=\frac{A}{r}\hat{\theta}$ where A is a constant.

a) Show that workdone due to displacement $d\vec{r}$ does not depend on r but only on the angle subtended. [2]

b) Find the total work done in each case for paths given below (for figure (i) the path is between points A and B). What property gives the force such peculiar behaviour? [1+1+2+1]

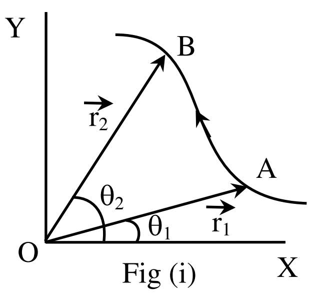


Fig (i)

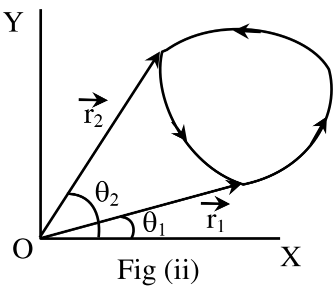


Fig (ii)

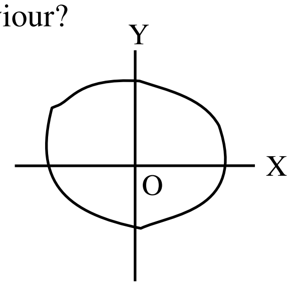
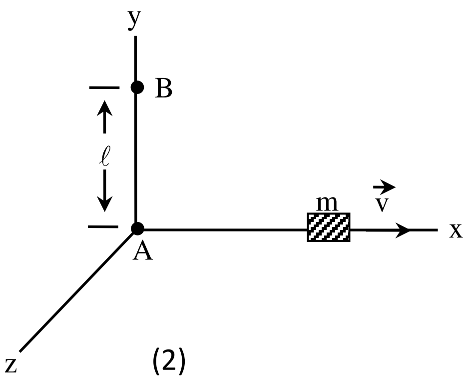


Fig (iii)

c) A particle of unit mass is projected vertically upwards in the gravitational field of earth, with initial velocity v_0. The medium offers a resistance to motion proportional to the square of the instantaneous speed v, i.e, $R=-kv^2(k>0)$. Write down the equation of motion and calculate the time of rise to the maximum height. [1+2]

9. a) A rocket is moving in free space far away from any gravitating matter by burning fuel at constant rate. What will be the velocity of the rocket at any time. [3]

b) Consider a block of mass m and negligible dimensions sliding freely in the x-direction with velocity $\vec{v}=v\hat{i}$, as shown in the sketch. What is its angular momentum $\vec{L}_A$ about the origin A and its angular momentum $\vec{L}_B$ about the origin B? [4]



c) A solid disk of mass M =2kg, radius R = 30 cm and moment of inertial I = $0\cdot09$ kgm^2 is mounted on a frictionless, horizontal axle. A light cord wrapped around the disk supports an object of mass m = $0\cdot5$ kg. Calculate the angular acceleration of the wheel, the linear acceleration of the object, and the tension in the cord. [3]

10. A particle of unit mass moves under the central force of the form $f(r) = -\frac{k}{r^2}$.

a) Plot the effective potential for this force law. What should be the nature of orbit? [2]

b) Solve the equation of motion and hence determine the nature of the orbit. [5]

c) Show that the Laplace-Runge-Lenz (LRL) vector denoted by $\vec{A} = \vec{p} \times \vec{L} - k\hat{r}$ is a constant of motion, where $\vec{p}$ is the linear momentum and $\vec{L}$ is the angular momentum. [3]

11. a) Deduce a relation between axial modulus (χ), Young's modulus (Y) and Poison's ratio (σ). [3]

b) Find the depression of a cantilever beam of uniform cross-section A and weight W, when loaded at the free end by a weight W_0. Hence find the period of oscillation when the free end of such loaded cantilever is further displaced and set free. [5+2]

12. a) What are the different types of forces exist in fluid statics? Establish the condition of equilibrium in fluid statics. [1+3]

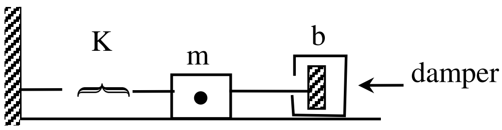
b) If water is flowing down a horizontal tube of circular cross-section of radius 1mm under pressure gradient of 10 dynes/cm^3, find the velocity of flow at a point $0\cdot5$ mm from the centre of the tube. Given that the coefficient of viscosity of water is $0\cdot01$ gmcm^{-1}sec^{-1}. [4]

c) Using Stoke's law in connection with the viscous drag experienced by a moving spherical body, find the terminal velocity of a rain-drop with radius 10^{-5}m falling through air. (co-efficient of viscosity of air = $1\cdot8\times10^{-5}$ decapoise). [2]

<u>Group – C</u>

(***Answer <u>any three</u> questions***) [3×10]

13. The physical system looks something like this :



The following observations have been made on this system :

- if the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.
- the viscous resistive force is equal to mg if the block moves with a certain known speed u.

a) Write the differential equation that describes the position of mass using m, b, K. [1]

b) Express the differential equation in terms of h and u. [2]

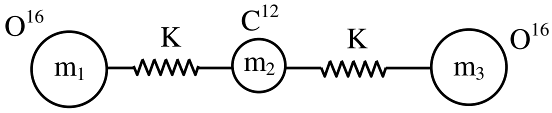
c) Answer the following questions for the special case that $u = 3\sqrt{gh}$.

i) What is the angular frequency of the damped oscillations? [3]

ii) After what time, expressed as a multiple of $\sqrt{\frac{h}{g}}$, is the energy down by a factor of $\frac{1}{e}$? [2]

iii) What is the Q of this oscillator? [2]

14. The CO_2 molecule can be linked to a system made up of a central mass m_2 connected by equal springs of spring constant K to two masses m_1 and m_3 (with $m_1 = m_3$) as shown :



a) Set up the equation of motion for m_1, m_2 and m_3. [3]

b) Solve the equations for the normal mode in which three masses are translated in the same direction and they do not oscillate. (Hint : Consider the case : eigenvalue $\lambda = 0$ for characteristic equation). [7]

15. a) Set up differential equation of a wave. [3]

b) Whether $y = e^{\frac{(vt-z)^2}{b^2}}$, b = constant, represents a wave or not? [2]

c) Let $y = \exp[(-az^2 - bt^2 - 2\sqrt{ab}zt)]$.

i) In what direction is the wave travelling? [1]

ii) What is the wave speed? [2]

iii) Sketch the wave for time t = 0 and for time t = 3 sec when a = 144/cm^2 , b = 9/sec^2. [2]

16. a) Define group velocity and phase velocity. Find a relation between them. [2+2]

b) "Velocity of the group of matter waves is just equal to the particle velocity" — prove it. [3]

c) For gravity waves in a liquid the phase velocity c depends on the wavelength λ according to the formula $c = A\sqrt{\lambda}$, A being a constant. Show that the group velocity is half of the phase velocity. [3]

17. a) Derive an expression for the velocity of plane longitudinal wave along a solid rod. Mention the assumption made. [5]

b) For a stretched string of length ℓ the displacement is given by

$$y(x,t) = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{\ell} \cos\,(\omega_n t - \phi_n)$$

where the symbols have their usual significance. Show that the total energy of the string is $E = \frac{M}{4}\sum \omega_n^2 c_n^2$ where M is the mass of the string. [5]

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